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Steering compensation for high-performance motorcycles

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Abstract— **This paper introduces the idea of using a mechanical steering compensator to influence the dynamic behaviour of a high-performance motorcycle. The compensator is seen as a possible replacement for a conventional steering damper, and comprises a network of a spring, a damper and a less familiar component called the inerter. The inerter was recently introduced to allow the synthesis of arbitrary passive mechanical impedances, and finds a new potential application in the present work. The approach taken here to design the compensator is based on classical Bode-Nyquist frequency response ideas. The vehicle study involves computer simulations, which make use of a state-of-the-art motorcycle model whose parameter set is based on a Suzuki GSX-R1000 sports machine. The study shows that it is possible to obtain significant improvements in the dynamic properties of the primary oscillatory modes, known as "wobble" and "weave", over a full range of lean angles, as compared with the standard machine fitted with a conventional steering damper.**

I. INTRODUCTION

The dynamics of motorcycles and their potential modes of instability have been studied for decades. In the case that one or more of these modes is stable, but lightly damped, the potential exists for undesirable responses to uneven road surfaces. Early research on motorcycle dynamics was confined to the relatively simple case of small perturbations from straight running, see [1–3] and the references therein. In later work these models were extended to include small perturbations from a steady-state cornering condition [4–9]. It is clear from these studies that under some operating conditions some of the machine's modes can be lightly damped, or even unstable. It is also clear that the lightly damped modes can be excited by road undulations [10]. The latter paper refers to several real-life incidents in which resonant forcing type phenomena ended unhappily for the rider. In addition to this theoretical work, motorcycle oscillations have been widely studied via measurement programmes [11–25].

The main lateral oscillations in two-wheeled vehicles are "weave" and "wobble". In straight running, the weave mode is well damped at moderate speeds, but becomes less so as the speed increases. The natural frequency rises from zero at very low speed to somewhere in the range $2 - 4$Hz, depending on the mass and size of the machine; the lower frequencies corresponding to heavier motorcycles. The only properly documented wobble oscillations involve moderate speeds, although there are many anecdotal accounts of wobble at high speeds [26]. The frequency of the wobble mode is relatively independent of speed, and is governed primarily by the mechanical trail, the front tyre cornering stiffness and the front frame steer inertia. The wobble mode's frequency is normally in the range $6 - 9$Hz. Stiff framed machines, being prone to wobbling at high speed, often rely on a steering damper for satisfactory wobble mode damping. Normally, however, a steering damper will destabilise the high-speed weave mode. In cornering, the above lateral modes and the in-plane modes associated with tyre deflections and suspension motions become coupled, as was first shown in any detail by Koenen [4]. The motorcycle becomes prone to resonant forcing via regular road undulations when the displacement forcing they produce is tuned to lightly damped modal frequencies of the machine. Moderate roll angles appear to represent the worst case conditions [10].

The free steering system of a single-track vehicle is essential to its stability and control behaviour [7]. It enables the machine to self-steer, to some extent, and it allows the rider to operate in free-control, or provide a steering torque input for control purposes. The question naturally arises as to whether there are better ways of influencing the self-steering action than via the use of a simple damper. The primary aim of this research is to investigate the benefits that can be derived from using a more general mechanical network consisting of springs, dampers and inerters, as compared with a conventional steering damper.

The paper is organised as follows. Section II reviews the nature and properties of the inerter which is still a relatively unfamiliar and novel mechanical component. In Section III the background to the motorcycle model is described. Some of the important characteristics of the reference motorcycle-rider system are described in Section IV. A frequency response based design procedure for a simple mechanical network is given in Section V and its performance is evaluated. Conclusions are drawn in Section VI.

II. THE INERTER

A two-terminal mechanical element called the inerter was introduced in [27] with the property that the (equal and opposite) force applied at the terminals is proportional to the relative acceleration between them. In the notation of Figure 1, $F = b(\dot{v}_1 - \dot{v}_2)$, where the constant of proportionality b is called the *inertance* and has the units of kilograms. In order to be practically useful, the device should have a small mass (relative to b) and its inertance should be adjustable independently of the mass. One way to make such a device is illustrated in Figure 2, where a plunger, which is constrained to translate relative to a

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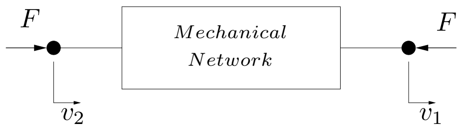

Fig. 1. Free-body diagram of a two-terminal mechanical element with force-velocity pair (F, v) in which $v = v_1 - v_2$.

housing, drives a flywheel via a rack and pinion, and gears [27]. In such cases the value of the inertance b is easy to compute as follows: if the device gives rise to a flywheel rotation of α radians per meter of relative displacement between the terminals, then the inertance of the device is given by $b = J\alpha^2$ where J is the flywheel's moment of inertia (when other masses and inertia effects are neglected). Various embodiments of inerters are described in [28] and several prototype devices have been built and tested in the Engineering Department at Cambridge University.

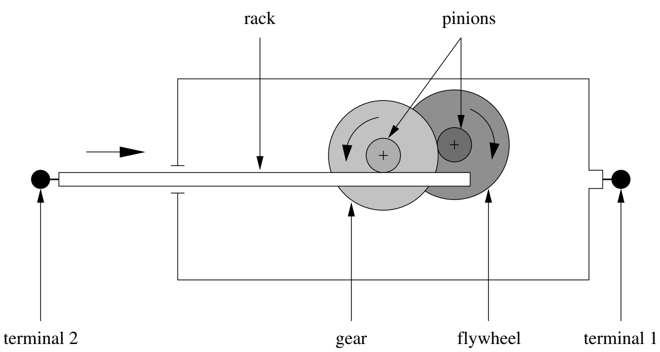

Fig. 2. Schematic of a mechanical model of an inerter.

A rotational version of the inerter can also be defined: namely, a device where the (equal and opposite) torque applied at each terminal (which can be separately rotated) is proportional to the relative angular acceleration between the terminals. The *inertance* of such a device is measured in kg m^2. For an ideal device of this type in which there is a gear ratio of n between angular rotations of the terminals and a flywheel with moment of inertia J, the inertance is given by Jn^2. Embodiments in pure rotational form can be devised, for example, by making use of epicyclic gears [28]. An alternative in the present context would be to use a linear inerter in a connecting link between the steering assembly and the main body.

One of the principal motivations for the introduction of the inerter in [27] was the synthesis of passive mechanical networks. It was pointed out that the standard form of the electrical-mechanical analogy (in which the spring, mass and damper are analogous to the inductor, capacitor and resistor) was restrictive for this purpose, because the mass element effectively has one terminal connected to ground. With the inerter replacing the mass, the full power of electrical circuit synthesis theory can be translated over to mechanical networks. This paper does not seek to exploit this theory in full. Instead, a single compensator is designed using classical Bode-Nyquist frequency response ideas.

III. MOTORCYCLE MODEL

The dynamics of motorcycles and the model used to represent them involve three translational and three rotational freedoms of the main frame, a steering freedom associated with the rotation of the front frame relative to the main frame and the influences of spinning road wheels. The mathematical model employed here also accommodates front and rear suspension freedoms, frame twisting, aerodynamic forces and moments and rolling of the rider's upper body relative to the main frame. The forces and moments associated with the tyres are modelled using "magic formulae" whose parameters have been optimized to fit measured rig data [29–32]. A motorcycle model incorporating all of the above features is described in detail elsewhere [5, 8, 9]. The parameters used in the model derive from laboratory experiments conducted on a contemporary commercially available sports machine, the Suzuki GSX-R1000. The effects of a steering damper, or a more general steering compensator, can be incorporated via the differential equations describing it. For the particular purposes of the study presented here, the steering compensation system is separated from the rest of the model in the generalized regulator feedback structure [33] shown in Figure 3. See [34] for a similar use of a control systems paradigm applied to vehicle suspension. While this figure shows a frequency domain model of the linearized system, it is equally applicable to nonlinear time domain models. The steering compensator appears as $K(s)$. Repeated reference will be made to Nyquist diagrams and the Nyquist criterion of the open-loop system $-sK(s)P_{22}(s)$, in which $P_{22}(s)$ maps the steering torque $T_s(s)$ into the steering angular velocity $s\delta(s)$.

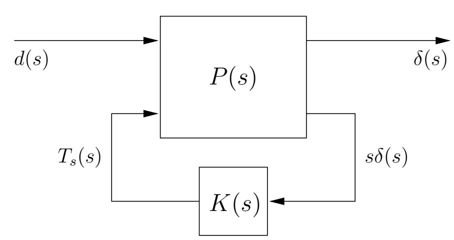

Fig. 3. $P(s)$ is the linearised motorcycle model and $K(s)$ is the steering compensator. The signal $d(s)$ represents road disturbances, $T_s(s)$ is the steering torque and $\delta(s)$ is the steering angle.

IV. CHARACTERISTICS OF THE STANDARD MACHINE

The important oscillatory modes associated with "wobble" and "weave" are illustrated in the root-locus diagrams of Figures 4 and 5. In both cases the machine's forward speed is the swept parameter. Figure 4 corresponds to the straight running machine without a steering damper. It can be seen from this diagram that the wobble modal frequency varies between 47 and 57 rad/s, while the weave mode's

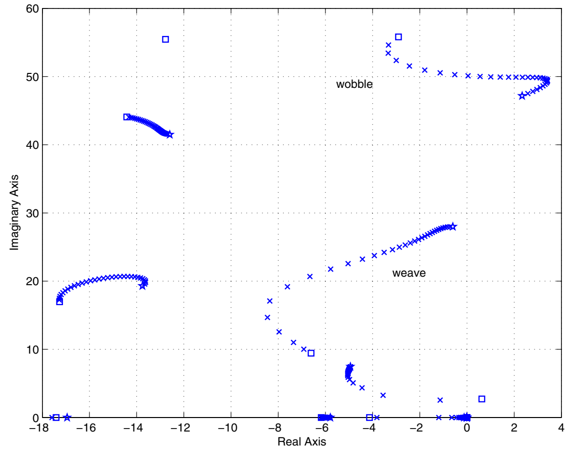

Fig. 4. Root-locus plot for the straight running nominal motorcycle with speed the varied parameter. No steering damper is fitted. The speed is increased from 5 m/s (□) to 85 m/s (⋆).

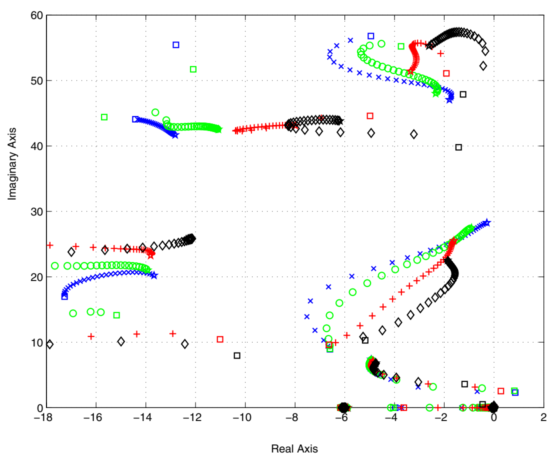

Fig. 5. Root-locus plots for: Straight running (×), 15 deg (○), 30 deg (+) and 45 deg (◇) of roll angle with speed the varied parameter. The nominal steering damper is fitted. The speed is increased from 6 m/s (□) to 75 m/s (⋆) for the 45 deg roll angle case and from 5 m/s (□) to 75 m/s (⋆) for each of the other cases.

resonant frequency varies between 10 and 28 rad/s. In Figure 5 the nominal steering damper is fitted and speed sweeps are carried out for four values of lean angle.

It can be seen from Figure 4 that the damping of the wobble mode decreases with increased speed, and when there is no steering compensation it becomes unstable at approximately 25 m/s. With further speed increases up to approximately 62 m/s, the real part of the wobble mode increases and then reduces again. By comparing Figures 4 and 5, it can be seen that the steering damper increases effectively the damping of the wobble mode, while reducing that of the weave mode.

Figure 5 also shows that increased values of roll angle tend to increase the high-speed weave mode damping. Since the coupling between the in-plane and out-of-plane dynamics increases with roll angle, one expects the weave mode vulnerability of the machine to road displacement forcing to maximize at an intermediate value of approximately 15 deg [10]. It can also be seen from Figure 5 that for roll angles of up to 30^o, the high speed wobble mode damping increases with roll angle. Further increases in roll angle then destabilize this mode. At low speeds the wobble mode damping decreases monotonically with roll angle and the vulnerability of this mode is worst at low speed and high roll angles.

The open-loop linearized motorcycle model can also be used to generate the Nyquist diagram shown in Figure 6. As will be appreciated from Figure 4, at this high-speed straight-running operating point the open-loop plant has two unstable poles corresponding to the wobble mode. In the case of a steering damper as the compensator in the feedback loop of Figure 3, $K(s)$ becomes a constant, K, say. It follows from the well-known Nyquist criterion [35] that closed-loop stability requires N anticlockwise encirclements of the $-1/K$ point, where N is the number of the unstable poles of the open-loop system and K is the value of the steering damping. If the steering damper is

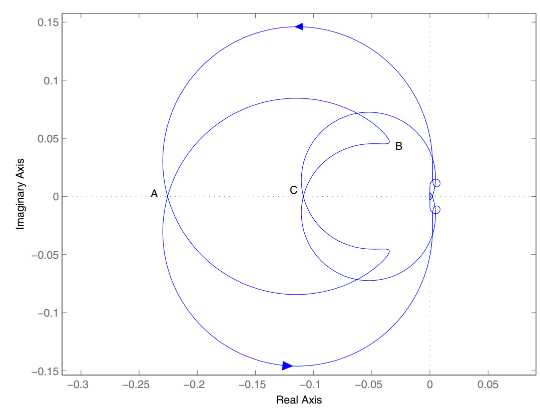

Fig. 6. Straight running Nyquist diagram for the open-loop motorcycle model at 75 m/s. The frequency at A is 47.6 rad/s, at B it is 33.8 rad/s and at C it becomes 28.4 rad/s.

set at a low value such that the $-1/K$ point is located at A, the system is on the stability boundary and will oscillate at 47.6 rad/s which is the wobble modal frequency. If the steering damping is now increased, two anticlockwise encirclement of the $-1/K$ point result and the machine will be stable. If the steering damping is increased further so that the $-1/K$ point is coincident with the point C in Figure 6, the machine will oscillate at 28.4 rad/s indicating that the weave mode is on the stability boundary. Finally, any further increases in the steering damping will render the machine unstable because the $-1/K$ point is not encircled at all. The nominal steering damper value is 6.944 Nm/rad/s thereby locating the $-1/K$ point at -0.144 in Figure 6. At 85 m/s the point C moves to the left, consistent with the tendency for the weave mode damping to reduce at very high speeds.

V. FREQUENCY RESPONSE DESIGN

A. Preliminary Observations

In order to develop design methodologies for passive steering compensators for the nominal motorcycle, the influences of the damper and the inerter, as isolated components, are briefly studied first. This will be done by investigating their effect on the wobble and weave mode damping/stability. It can be seen that the introduction of a steering damper improves the damping of the wobble mode. Figure 7 shows the effect of changes to the (nominal)

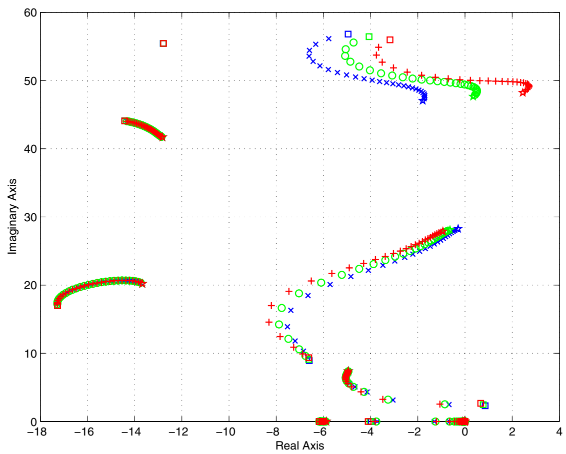

Fig. 7. Straight running root-loci with speed the varied parameter. The speed is increased from 5 m/s (□) to 75 m/s (⋆). × represents the nominal machine, ∘ refers to a steering damping decrease of 3 Nms/rad and + to a steering damping reduction of 6 Nms/rad.

steering damper parameter value. As the steering damper value is reduced, the wobble mode becomes unstable at high speed, while the high-speed weave mode damping increases slightly. The damper has almost no effect on the natural frequencies of either of these modes.

Figure 8 illustrates the effect on the machine's modal damping characteristics of introducing an inerter. It may be observed that the wobble mode natural frequency reduces as the value of inertance is increased. This is intuitively expected since the wobble mode chiefly involves rotation of the front frame assembly, so the change is similar to increasing its moment of inertia. It also reduces the wobble mode damping and increases that of the weave mode. When comparing Figures 7 and 8 one is drawn to the idea that an effective steering compensator should 'look like' an inerter at low frequencies in order to improve the damping of the weave mode, while taking on the mantel of a damper at higher frequencies in order to stabilize the wobble mode. This can be interpreted as a form of lead compensation.

B. Lead Network Design

The admittance function of the mechanical network in Figure 9 is given by

$$K(s) = c\frac{s(s+k/c)}{s^2+(c/b)s+k/b}$$

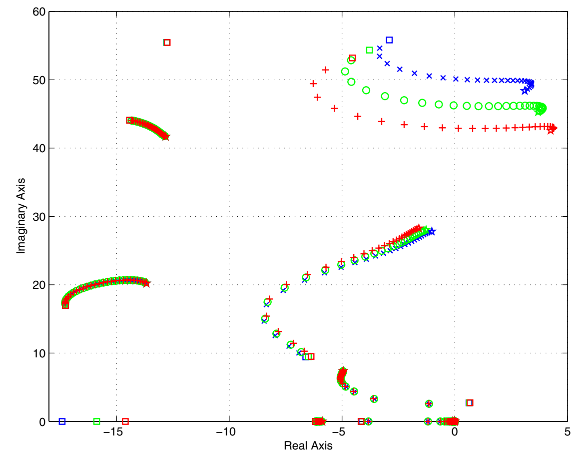

Fig. 8. Straight running root-loci with speed the varied parameter; the speed is increased from 5 m/s (□) to 75 m/s (⋆). × represents the nominal machine without a steering damper, ∘ represents the effect of an inertance of 0.1 kgm^2 and + the influence of an inertance of 0.2 kgm^2.

which can be re-parameterized using the substitution $\omega_n = \sqrt{k/b}$ and $\zeta = c/(2\sqrt{kb})$ as

$$K(s) = c\frac{s(s+\frac{\omega_n}{2\zeta})}{s^2+2\zeta\omega_n s+\omega_n^2}. \quad (1)$$

Its frequency response characteristics are illustrated in Figure 10, which has been normalized to $\omega_n = 1$. As

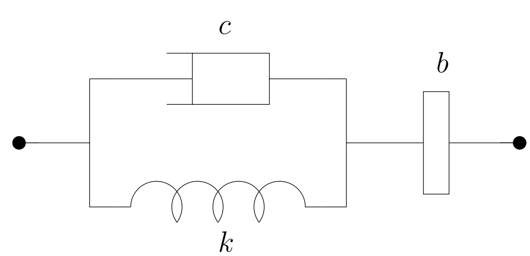

Fig. 9. Mechanical lead compensator comprising a spring k, damper c and inerter b.

expected the resonant peak becomes more and more pronounced as the value of ζ is reduced. At the same time one observes an increasingly rapid phase transition in the neighbourhood of ω_n and the phase lead at ω_n, ϕ_n, decreases as well. It is easy to show that the phase lead at $s = j\omega_n$ is equal to

$$\phi_n = \arctan(2\zeta).$$

We now turn to the development of a systematic design procedure for the lead network. A careful examination of the Nyquist plot in Figure 6 reveals that, for this high-speed straight running condition, it is advantageous for the steering compensator to introduce phase lead up to the cusp at point B (33.8 rad/s) and proportional gain thereafter. This observation is consistent with the notion that weave requires

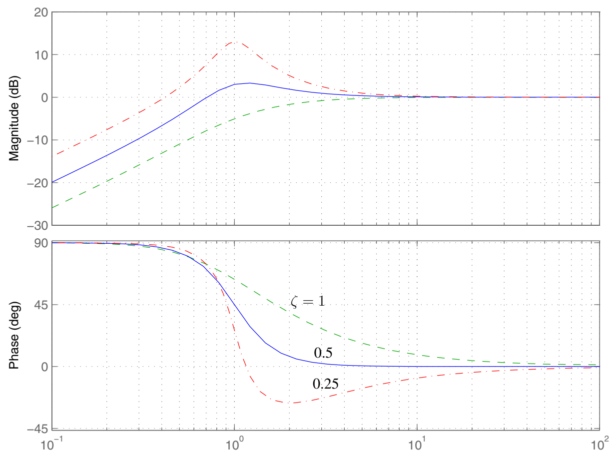

Fig. 10. Frequency responses for $\frac{s(s+\frac{\omega_n}{2\zeta})}{s^2+2\zeta\omega_n s+\omega_n^2}$ with resonant frequency $\omega_n = 1$ for three values of damping ratio ζ.

derivative action, while wobble requires proportional feedback. It was found that a value of $\omega_n = 33.8$ together with a damping ratio of $\zeta = 0.5$ gave a suitable result. The influence of this unity gain compensator is illustrated in Figure 11. As intended, the derivative action has moved the negative-axis crossing point associated with weave mode instability towards the origin and the crossing point linked to wobble to the left of the diagram. This "opens up" the interval over which two anti-clockwise encirclements of the $-1/K$ point can be achieved. In order to maximize the radius of a circle centred at -1, and which can be encircled twice by the Nyquist diagram, the damper coefficient was chosen to be $c = 1/0.174 = 5.77$. This places the -1 point at the mid point between the two negative real-axis crossing points. Given these values for c, ζ and ω_n, the b

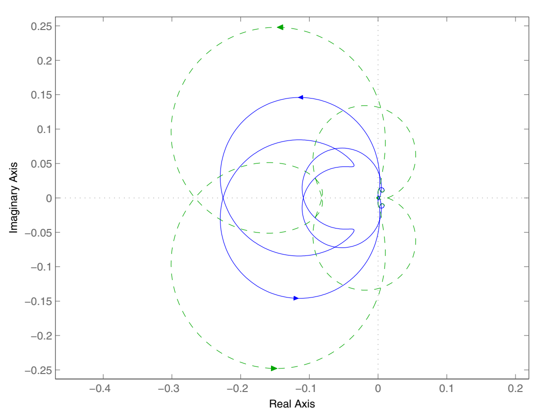

Fig. 11. Nyquist diagram for the straight-running open-loop motorcycle at a forward speed of 75 m/s. The solid line represents the nominal machine without a steering damper and the dashed line the compensated system using the network given in Figure 9 ($\omega_n = 33.8$, $\zeta = 0.5$, $c = 1$).

and k parameters can be found via back substitution using:

$$b = \frac{c}{2\zeta\omega_n},$$
$$k = \frac{c\omega_n}{2\zeta}.$$

This gives the network parameter values listed in Table I. The effect of the simple mechanical network illustrated in

TABLE I

PARAMETERS FOR SIMPLE MECHANICAL NETWORK GIVEN IN FIGURE 9.

c	ζ	ω_n
5.744	0.5	33.8
c	b	k
5.744	0.1699	194.14

Figure 9, with the parameters given in Table I, is shown in Figure 12. Although this design is based on a single high-speed straight-running linearized model, it is evident from these root-locus plots that, in comparison with the nominal machine behaviour given in Figure 5, substantial improvements in the damping of the weave mode, under all operating conditions, have been achieved. Greatly improved wobble-mode damping has also been obtained. The high-roll-angle (45 deg) case is worthy of particular note.

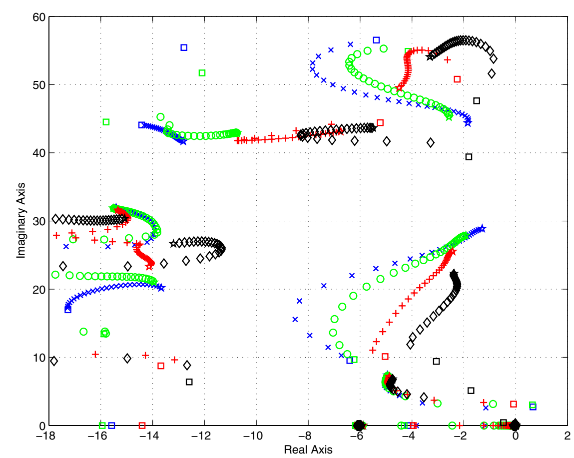

Fig. 12. Root-locus plots for: Straight running (×), 15 deg (○), 30 deg (+) and 45 deg (◇) with speed the varied parameter for the compensated machine with the hand designed lead network illustrated in Figure 9. The speed is increased from 6 m/s (□) to 75 m/s (⋆) for the 45 deg roll-angle case and from 5 m/s (□) to 75 m/s (⋆) for each of the other cases.

VI. Conclusions

This paper has introduced the idea of replacing a conventional steering damper with a mechanical network comprising a spring, damper and inerter [27] on a high-performance motorcycle. The study has used an advanced motorcycle simulation model [5, 8, 9] to demonstrate that this can lead to clear performance benefits in wobble and weave mode damping. For the particular mechanical network introduced a design methodology has been developed based on classical frequency response control system design ideas [35].

This study was prompted by the oft reported poor performance of high-powered sports motorcycles operating at high speed. Figure 5 shows that the nominal machine is vulnerable to high-speed weave, particularly under straight running conditions, and low speed wobble, particularly at high roll angles. Preliminary studies presented in Section V-A showed that a steering damper is an effective means of damping the wobble mode, but that it can have a deleterious effect on the weave mode. These studies also show that a steering inerter can improve the damping of the weave mode. These observations motivated the study of the simple mechanical network presented in Figure 9. This network "looks like" a damper above the weave mode frequency band, while adopting the mantel of an inerter at lower frequencies. This may be interpreted as a form of mechanical lead compensation. A simple frequency response design procedure is presented for this network. In a first design step, the compensation network's natural frequency and damping ratio are chosen. In a second step, which is conducted after the compensated Nyquist diagram is plotted, the network's damper value is selected. As is demonstrated in Figure 12, this network greatly improves the motorcycle's performance characteristics as compared with the nominal machine.

In the general area of passive mechanical compensator implementation, several issues remain outstanding. These include the integrated fabrication of the mechanical network selected, the selection of optimal gear ratios for any inerters used and the correct dimensioning of the device in order that it is robust enough to withstand the wear and tear of normal usage, while not being needlessly unwieldy. The steering compensator also has to be unobtrusively fitted to the machine once it has been made. These are all topics for further study.

REFERENCES

[1] R. S. Sharp, "The stability and control of motorcycles," *Jour. Mech. Eng. Sci.*, vol. 13, no. 5, pp. 316–329, 1971.

[2] R. Sharp, "Vibrational modes of motorcycles and their design parameter sensitivities," in *Vehicle NVH and Refinement, Proc Int Conf.* Birmingham: Mech. Eng. Publications, London, 3–5 May 1994, pp. 107–121.

[3] D. J. N. Limebeer, R. S. Sharp, and S. Evangelou, "The stability of motorcycles under acceleration and braking," *Proc. Inst. Mech. Eng.*, vol. 215, no. C9, pp. 1095–1109, 2001.

[4] C. Koenen, "The dynamic behaviour of motorcycles when running straight ahead and when cornering," Ph.D. dissertation, Delft University of Technology, 1983.

[5] R. S. Sharp and D. J. N. Limebeer, "A motorcycle model for stability and control analysis," *Multibody System Dynamics*, vol. 6, no. 2, pp. 123–142, 2001.

[6] V. Cossalter and R. Lot, "A motorcycle multi-body model for real time simulations based on the natural coordinates approach," *Vehicle System Dynamics*, vol. 37, no. 6, pp. 423–447, 2002.

[7] R. S. Sharp, "Stability, control and steering responses of motorcycles," *Vehicle System Dynamics*, vol. 35, no. 4–5, pp. 291–318, 2001.

[8] R. S. Sharp, S. Evangelou, and D. J. N. Limebeer, "Improved modelling of motorcycle dynamics," in *ECCOMAS Thematic Conference on Advances in Computational Multibody Dynamics*, J. Ambrósio, Ed., Lisbon, July 1–4 2003, MB2003-029 (CD-ROM).

[9] ——, "Advances in the modelling of motorcycle dynamics," *Multibody System Dynamics*, 2004, in press.

[10] D. J. N. Limebeer, R. S. Sharp, and S. Evangelou, "Motorcycle steering oscillations due to road profiling," *Transactions of the ASME, Journal of Applied Mechanics*, vol. 69, no. 6, pp. 724–739, 2002.

[11] E. Döhring, "Steering wobble in single-track vehicles," *ATZ*, vol. 58, no. 10, pp. 282–286, 1956.

[12] D. J. Eaton, "Man-machine dynamics in the stabilization of single-track vehicles," Ph.D. dissertation, University of Michigan, 1973.

[13] Y. Watanabe and K. Yoshida, "Motorcycle handling performance for obstacle avoidance," in 2^{nd} *Int. Congress on Automotive Safety*, San Fransisco, 1973.

[14] G. Jennings, "A study of motorcycle suspension damping characteristics," *SAE 740628*, 1974.

[15] G. E. Roe and T. E. Thorpe, "A solution of the low-speed wheel flutter instability in motorcycles," *Jour. Mech. Eng. Sci.*, vol. 18, no. 2, pp. 57–65, 1976.

[16] M. K. Verma, "Theoretical and experimental investigation of motorcycle dynamics," Ph.D. dissertation, University of Michigan, 1978.

[17] J. S. McKibben, "Motorcycle dynamics – fact, fiction and folklore," *SAE 780309*, 1978.

[18] A. Aoki, "Experimental study on motorcycle steering performance," *SAE 790265*, 1979.

[19] D. H. Weir and J. W. Zellner, "Experimental investigation of the transient behaviour of motorcycles," *SAE 790266*, 1979.

[20] B. Thomson and H. Rathgeber, "Automated systems used for rapid and flexible generation of system models exemplified by a verified passenger car and a motorcycle model," in *Dynamics of Vehicles on Roads and on Railway Tracks*, J. K. Hedrick, Ed. Swets and Zeitlinger, Lisse, 1984, pp. 645–654.

[21] B. Bayer, "Flattern und pendeln bei krafträdern," *Automobil_Industrie*, vol. 2, pp. 193–197, 1988.

[22] T. Takahashi, T. Yamada, and T. Nakamura, "Experimental and theoretical study of the influence of tires on straight-running motorcycle weave response," *SAE 840248*, 1984.

[23] W. M. Otto, "Effect of motorcycle accessories on stability," in *Proc. International Motorcycle Safety Conference*. Linthicum MD 21090, Washington DC: Motorcycle Safety Foundation, May 1980, pp. 1560–1581.

[24] M. Sugizaki and A. Hasegawa, "Experimental analysis of transient response of motorcycle rider systems," *SAE 881783*, 1988.

[25] F. Biral, D. Bortoluzzi, V. Cossalter, and M. Da Lio, "Experimental study of motorcycle transfer functions for evaluating handling," *Vehicle System Dynamics*, vol. 39, no. 1, pp. 1–26, 2003.

[26] T. Foale. (2002) Motorcycle handling and chassis design – the art and science. [Online]. Available: http://www.tonyfoale.com

[27] M. C. Smith, "Synthesis of mechanical networks: The inerter," *IEEE Trans. Automatic Control*, vol. 47, no. 10, pp. 1648–1662, October 2002.

[28] ——, "Force-controlling mechanical device," Patent pending, International Application No. PCT/GB02/03056, 4 July 2001.

[29] E. de Vries and H. Pacejka, "The effect of tyre modeling on the stability analysis of a motorcycle," in *Proc. AVEC'98*. Nagoya: SAE of Japan, 1998, pp. 355–360.

[30] ——, "Motorcycle tyre measurements and models," in *Proc. 15th IAVSD Symposium on the Dynamics of Vehicles on Roads and on Tracks*, L. Palkovics, Ed., Budapest Hungary, August 25–29 1997, suppl. Vehicle System Dynamics, **28**, 1998, 280–298.

[31] Y. Tezuka, H. Ishii, and S. Kiyota, "Application of the magic formula tire model to motorcycle maneuverability analysis," *JSAE Review*, vol. 22, pp. 305–310, 2001.

[32] H. Pacejka, *Tyre and Vehicle Dynamics*. Oxford: Butterworth Heinemann, 2002.

[33] M. Green and D. J. N. Limebeer, *Linear Robust Control*. Englewood Cliffs, New Jersey 07632: Prentice Hall, 1995.

[34] C. Papageorgiou and M. C. Smith, "Positive real synthesis using matrix inequalities for mechanical networks: application to vehicle suspension," *43rd IEEE Conference on Decision and Control*, 14-17 December 2004.

[35] R. C. Dorf and R. H. Bishop, *Modern Control Systems*. Upper Saddle River, NJ: Prentice Hall, 2001.