ε-Near-Zero Graded Index Structure as a Bi-concave Metallic Lens Using Stacked Rectangular Near Cut-Off Waveguides

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***Abstract*—In this paper a slab of stacked near cut-off rectangular waveguides emulating an ε-near-zero (ENZ) artificial material is designed to behave as a bi-concave symmetric lens. First, a rectangular narrow hollow waveguide is simulated in order to show how such device can be used to mimic an ENZ medium. Afterwards, a Graded Index ENZ structure emulating a symmetric 7λ_0 input and output focal length converging bi-concave lens is designed working at 100 GHz.**

***Index Terms*—Graded index lens, metallic lens, ENZ artificial materials.**

I. INTRODUCTION

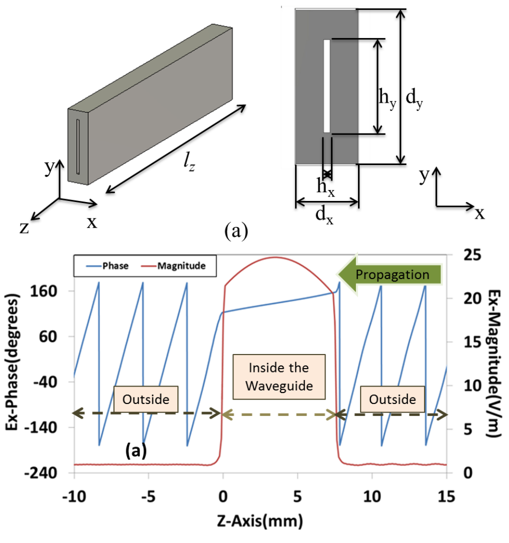
Permittivity (ε)-near-zero (ENZ) artificial materials have been studied since last decade due to their unconventional features such as tunneling, energy squeezing and supercoupling [1]-[4]. Some applications have been proposed using these man-made materials such as beam shaping [3], nanocircuits [5] and sensing [6]. Moreover, recent research has shown that narrow hollow waveguides working near the cut-off frequency can be considered as a possible realization of an ENZ medium due to the small phase progression allowed inside them [1].

In this work, we propose a graded index ENZ structure emulating a bi-concave symmetric lens behavior based on [1]-[4]. The ENZ graded index lens is made by an arrangement of narrow hollow rectangular waveguides working near the cut-off frequency of the fundamental mode TE_{01}. The small phase variation inside each waveguide is used to generate the desired wavefront at the output of the structure focusing an incoming concave wavefront.

II. DESIGN AND SIMULATION RESULTS

To begin with, a single narrow hollow rectangular waveguide with h_y and h_x as the hollow dimensions is presented in Fig. 1(a). The waveguide is designed to work near the cut-off frequency of the dominant mode TE_{01} for $f \approx 100$ GHz ($\lambda_0 \approx 3$mm). Based on this, its dimensions are $d_x = 0.6$mm, $d_y = 2$mm, $h_x = 0.06$mm $h_y = 1.5$mm and $l_z = 2.5\lambda_0$.

The numerical analysis is made using the finite-integration-technique software CST Microwave Studio™. By placing top and bottom magnetic walls and left and right electric walls, the single waveguide is infinitely replicated along y and x axes since the electric field lies on the xz-plane. With these considerations, simulation results of the electric field magnitude and phase outside and inside the waveguide along z-axis are presented in Fig. 1(b). It shows that the magnitude of the electric field (E_x) is greater inside the waveguide and the resulting tunneling and supercoupling effects [2]-[4] can be observed due to the nearly uniform phase and amplitude distribution across the narrow hollow. Moreover, it can be observed that the phase progression inside the waveguide is slower that outside where it changes more rapidly.



(a)

(b)

Figure 1. (a) Representation of the narrow hollow waveguide: perspective (left) and front (right) views and (b) simulation results: E_x-field-magnitude (red line) and phase (blue line) along z-axis.

Based on this analysis, the graded index ENZ structure is made by placing side by side an array of 51 narrow hollow rectangular waveguides. The schematic representation of the slab is shown in Fig. 2(a), where md_x is the distance between

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the central waveguide used as reference and each one of the other members of the array, L_1 and L_2 represent the source position and the desired focal length at the output of the lens, respectively.

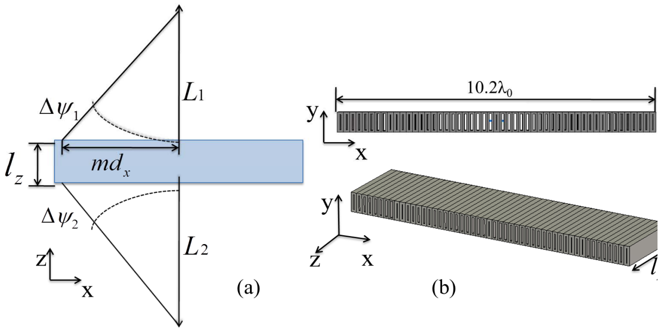

Figure 2. (a) Geometrical representation of the behavior of the phase delay of the lens and (b) representation of the graded index ENZ structure which emulates a bi-concave lens: front (top) and perspective (bottom) views.

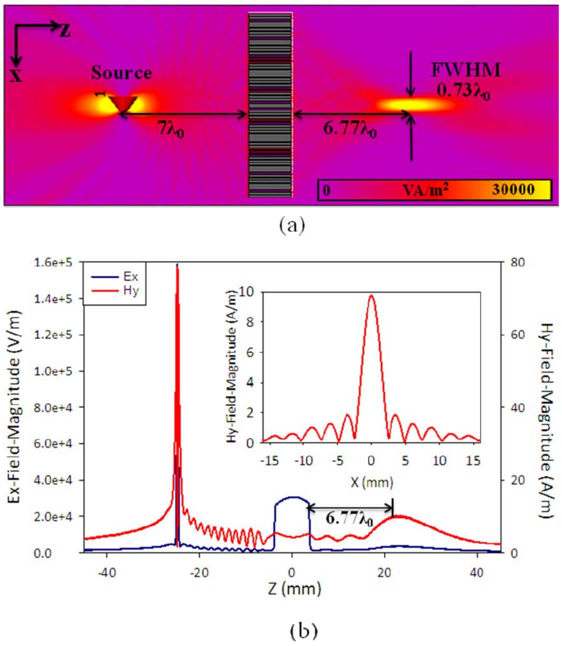

Figure 3 Simulation results of the bi-concave graded index ENZ flat lens with a designed symmetric focal length of $7\lambda_0$: (a) Power flow and (b) E_x-field (blue line) and H_y-field (red line) along z-axis, (inset) H_y-field at the output focal length along x-axis.

The phase delay difference ($\Delta\psi^{(m)}$) between each waveguide and the central one is obtained using (1) where, β_0 is the propagation constant of the reference waveguide placed at the center of the slab, k_0 is the wave number in free-space at the operation frequency, t is the waveguide dimension at z-axis and n is an integer number (n=1,2,3…). Note that the condition $L_1=L_2=L$ has been used in order to obtain a symmetric behavior of the lens. The phase delay that should be introduced by each waveguide is properly tuned following (2) where the hollow dimension h_y and the propagation constant of the waveguide are related.

$$\Delta\psi^{(m)} = \beta^{(m)} l_z = \beta_0 l_z - 2k_0\left[\sqrt{(L)^2+(md_x)^2} - L\right] + 2\pi n \quad (1)$$

$$\beta^{(m)} = k_0\sqrt{1-\left(\pi/k_0 h_y^{(m)}\right)^2} \quad (2)$$

The dimensions of the reference waveguide are the same used for the narrow hollow waveguide presented in Fig. 1(a) for a design frequency of $f \approx$ 100GHz and $L_1=L_2=L=7\lambda_0$=21mm.

The schematic of the designed graded index ENZ structure is presented in Fig. 2(b) and simulation results are shown in Fig. 3. As it can be observed in Fig. 3(a), where the power flow along the E-plane is depicted, the full width at half maximum (FWHM) obtained is $0.73\lambda_0$ and the focal length at the output of the lens is obtained at 20.3mm = $6.77\lambda_0$ which represents a small deviation of 3.3% from the designed value ($7\lambda_0$). The lensing performance together with the electric field enhancement within the ENZ slab is seen in Fig. 3(b) where the electric (E_x) and magnetic (H_y) field magnitudes along z-axis have been plotted. Moreover, the sinc-like function described by the H_y at the focal plane (inset of Fig. 3(b)) suggests that the slab performs a discrete Fourier transform as any dielectric lens [7]. These simulations have been validated independently with Comsol Multiphysics$^{\text{TM}}$. For instance, Power Flow distribution computed with Comsol is displayed in Fig. 4, confirming the lensing operation of the ENZ graded index slab.

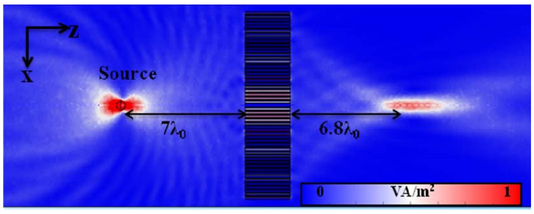

Figure 4. Power flow distribution of the bi-concave graded index ENZ flat lens using Comsol Multiphysics™.

III. CONCLUSIONS

In this work, a bi-concave lens emulated by a graded index ENZ metallic slab has been shown. The structure was designed by an array of stacked waveguides working near cut-off with a symmetric front and back focal length of $7\lambda_0$. Simulation results show that the focus is obtained at the output of the structure with a FWHM value of $0.73\lambda_0$. Furthermore, the focal length at the output of the ENZ slab is located at $6.77\lambda_0$ with a slight variation of 3.3% from the designed value. These results could find applications for novel lenses.

REFERENCES

[1] M. Silveirinha, and N. Engheta, “Tunneling of electromagnetic energy through subwavelength channels and bends using ε near-zero materials,” Phys. Rev. Lett., vol. 97, no. 15, pp. 157403-1-4, August 2006.

[2] M.G. Silveirinha, and N. Engheta, “Theory of supercoupling, squeezing wave energy, and field confinement in narrow channels and tight bends using ε near-zero metamaterials,” Phys. Rev. Lett., vol.76, no. 24, pp. 245109-1-17, December 2007.

[3] A. Alù, M. G. Silveirinha, A. Salandrino, and N. Engheta, “Epsilon-near-zero metamaterials and electromagnetic sources: Tailoring the

radiation phase pattern," Phys. Rev. B, vol. 75, no. 15, pp. 155410-1-13, April 2007.

[4] B. Edwards, A. Alù, M.E. Young, M. Silveirinha, and N. Engheta, "Experimental verification of epsilon near-zero metamaterial coupling and energy squeezing using a microwave waveguide," Phys. Rev. Lett., vol. 100, no. 3, pp. 033903-1-4, January 2008.

[5] N. Engheta, "Circuits with light at nanoscales: Optical nanocircuits inspired by metamaterials," Science, vol. 317, no. 5485, pp. 1698-1702, September 2007.

[6] A. Alù, and N. Engheta, "Dielectric Sensing in ε-Near-Zero Narrow Waveguide Channels," Phys. Rev. B, vol. 78, no. 4, pp. 045102-045106, July 2008.

[7] M. Navarro-Cia, M. Beruete, M. Sorolla, and N. Engheta, "Lensing System and Fourier Transformation using ε-near-zero (ENZ) metamaterials," Phys. Rev. B., vol. 86, no. 16, pp. 165130-1-6, October 2012.